The Fourier Transform of the Yukawa potential. The expression is:

$$\int \frac{Ze^2}{r} e^{-r/a} e^{i\bar{q}\cdot\bar{r}} \, dV$$

since this is a spherically symmetric potential we can make use of spherical coordinates with polar axis along $\bar{q}$:

$$Ze^2 \int_0^\infty \int_0^\pi \int_0^{2\pi} \frac{e^{-r/a}}{r} e^{iqr\cos\theta} r^2 \sin\theta \, d\varphi \, d\theta \, dr$$

$$= 2\pi Z e^2 \int_0^\infty \int_0^\pi \frac{e^{-r/a}}{r} e^{iqr\cos\theta} r^2 \sin\theta \, d\theta \, dr .$$ The angular

part: $\displaystyle\int_0^\pi e^{iqr\cos\theta} \sin\theta \, d\theta = \begin{bmatrix} u = \cos\theta \\ du = -\sin\theta \, d\theta \end{bmatrix}$

$$= -\int_1^{-1} e^{iqru} \, du = \frac{e^{iqr} - e^{-iqr}}{iqr} = \frac{2\sin qr}{qr}$$

$$\Rightarrow 2\pi Z e^2 \int_0^\infty e^{-r/a} \frac{2\sin qr}{q} \, dr = \frac{4\pi Z e^2}{q} \int_0^\infty e^{-r/a} \sin qr \, dr$$

$$= \Big[\int_0^\infty e^{-r/a} \sin qr \, dr = \mathrm{Im} \int_0^\infty e^{-r/a} e^{iqr} \, dr :$$

$$= \mathrm{Im} \left[-\frac{e^{-(1/a - iq)r}}{(1/a - iq)} \right]_0^\infty = \mathrm{Im} \frac{1}{(1/a - iq)}$$

$$= \operatorname{Im} \frac{1/a + iq}{1/a^2 + q^2} = \frac{q}{a^{-2} + q^2}$$, therefore

we get $\frac{4\pi Z e^2}{q} \cdot \frac{q}{q^2 + a^{-2}} = \frac{4\pi Z e^2}{q^2 + a^{-2}}$.

Summarized: $\int \frac{Ze^2}{r} e^{-r/a} e^{i\vec{q}\cdot\vec{r}} \, dV = \frac{4\pi Z e^2}{q^2 + a^{-2}}$