If we consider an electron (spin 1/2) scattering off a heavy nucleus, the scattering amplitude is given by

$f = A + 2B\bar{\nu}\cdot\bar{s}$ where $\bar{\nu} = \bar{n}\times\bar{n}'$ is the unit normal to the plane and $\bar{s}$ the spin operator. A & B are simply complex functions of θ.

But! Now we ask: what is the polarization of the scattered beam? Hence, we need to think about

a stream of electrons, not only one. To do this we introduce the density matrix. For the incoming beam we get:

$\rho_{in} = \frac{1}{2}\mathbb{1}$, encoding the fact that all

spin directions are equally probable. After scattering, the spin state transforms $\rho_{out} = f\rho_{in}f^{+}/\mathrm{Tr}(f\rho_{in}f')$

$= ff^{+}/\mathrm{Tr}(ff^{+})$ since $\rho_{in} = \frac{1}{2}\mathbb{1}$. The expectation value of any observable O in state ρ is $\langle O\rangle = \mathrm{Tr}(\rho O)$, which means that the expectation value of spin after scattering is $\langle\bar{s}\rangle = \mathrm{Tr}(\rho_{out}\bar{s}) = \mathrm{Tr}(ff^{+}\bar{s})/\mathrm{Tr}(ff')$.

The polarization vector is $\bar{P}' = 2\langle\bar{s}\rangle$ because we want $|\bar{P}| = 1$ in the fully polarized case.

This is why $$P' = \frac{2\,\mathrm{Tr}(ff'\bar{s})}{\mathrm{Tr}(ff')}.$$

computing $ff^\dagger$:

$$ff^\dagger = (A + 2B\bar{v}\cdot\bar{s})(A^\dagger + 2B^\dagger\bar{v}\cdot\bar{s})$$

$$= |A|^2\mathbb{1} + 2AB^\dagger\bar{v}\cdot\bar{s} + 2BA^\dagger\bar{v}\cdot\bar{s} + 4|B|^2(\bar{v}\cdot\bar{s})^2$$

For spin 1/2 $\bar{s} = \bar{\sigma}/2 \Rightarrow (\bar{v}\cdot\bar{s})^2 = \frac{1}{4}(\bar{v}\cdot\bar{\sigma})^2 = \frac{1}{4}|\bar{v}|^2\mathbb{1}$

where $|\bar{v}| = |n \times n'| = \sin\theta \Rightarrow |\bar{v}|^2 = \sin^2\theta$.

and $\mathrm{Re}(AB^\dagger) = \frac{1}{2}\, AB^\dagger + (AB^\dagger)^\dagger$.

$$ff^\dagger = (|A|^2 + |B|^2)\mathbb{1} + 2\,\mathrm{Re}(AB^\dagger)\cdot 2\bar{v}\cdot\bar{s}.$$

The trace $\mathrm{Tr}(ff^\dagger) = 2(|A|^2 + |B|^2)$ since $\mathrm{Tr}(\mathbb{1}) = 2$ and $\mathrm{Tr}(\bar{\sigma}) = 0$. For $\mathrm{Tr}(ff^\dagger\bar{s})$:

$$\mathrm{Tr}(ff^\dagger\bar{s}) = (|A|^2 + |B|^2)\mathrm{Tr}(\bar{s}) + 4\,\mathrm{Re}(AB^\dagger)\,\mathrm{Tr}((\bar{v}\cdot\bar{s})\bar{s})$$

The 2nd term, $\mathrm{Tr}((\bar{v}\cdot\bar{s})\bar{s}) = \frac{1}{4}\mathrm{Tr}((\bar{v}\cdot\bar{\sigma})\sigma_k)$

$$= \frac{1}{4}v_i\,\mathrm{Tr}(\sigma_i\sigma_k) = [\mathrm{Tr}(\sigma_i\sigma_k) = 2\delta_{ik}]$$

$= \frac{1}{4}v_i \cdot 2\delta_{ik} = \frac{1}{2}v_k$. Evidently, $\mathrm{Tr}(ff^\dagger\bar{s}) = 2\,\mathrm{Re}(AB^\dagger)\bar{v}$

Final result: $$\bar{P}' = \frac{2\,\mathrm{Tr}(ff^\dagger\bar{s})}{\mathrm{Tr}(ff^\dagger)} = \frac{2\cdot 2\,\mathrm{Re}(AB^\dagger)\bar{v}}{2(|A|^2 + |B|^2)}$$

$$= \frac{2\,\mathrm{Re}(AB^\dagger)}{|A|^2 + |B|^2}\bar{v}$$