Block II

With the spinor established, the hopping matrix eq. (1b) in TA reads:

$$(H_\nu)_{mn} = \sum_{s=\pm 1} \left(-t_\nu \delta_{n,m+s} + i\lambda_\nu v_m^{(s)} \cdot \sigma \, \delta_{n,m+2s} \right)$$

The first term, $-t_\nu \delta_{n,m+s}$ is the nearest-neighbour hopping: an electron moves from site m to site $m+1$ with amplitude $-t_\nu$. The index $s=+1$ is forward, $s=-1$ is backward. This is spin-independent, it acts as a scalar on the spinor, multiplying both components equally.

The second term, $i\lambda_\nu v_m^{(s)} \cdot \sigma \, \delta_{n,m+2s}$, is the next-nearest-neighbour spin-orbit coupling. It skips a site and involves σ, acting non-trivially in spin space. The operator $v_m \cdot \sigma$ has the same algebraic form as the rotation generators established in §3 §5, for any vector $\vec{n}$, the operator $\vec{n} \cdot \sigma$ generates rotations around the axis $\vec{n}$, but with $v_m^{(s)}$ as the axis, which is how we geometrically constructed the local curvature of the chain. An electron hopping from site m to $m+2s$ has its spinor rotated in a direction set by the local geometry of the chain at site m.