The sensing protocol

1. Initialize

$$|0\rangle$$

$\downarrow$ 2. Transform

$$|\psi_0\rangle = \hat{U}_a|0\rangle$$

$\downarrow$ 3. Evolve

$$|\psi(t)\rangle = \hat{U}_H(0,t)|\psi_0\rangle$$

$\downarrow$ 4. Transform

$$|\alpha\rangle = \hat{U}_B|\psi(t)\rangle$$

$\downarrow$ 5. Project

$$\text{"0"} \sim |\langle 0|\alpha\rangle|^2 \ \& \ \text{"1"} \sim |\langle 1|\alpha\rangle|^2$$

$\downarrow$ 6. Repeat

$$p = 1 - |\langle 0|\alpha\rangle|^2$$

$\downarrow$ 7. Estimate

$$\{p_i\} \Rightarrow V(t)$$

1. The quantum sensor is initialized into a known basis state.

2. The quantum sensor is transformed into the desired initial sensing state $|\psi_0\rangle = \hat{U}_a|0\rangle$.

3. The quantum sensor evolves under the Hamiltonian $\hat{H}$ for time t. At the end of the sensing period:

$$|\psi\rangle = \hat{U}_H(0,t)|\psi_0\rangle = c_0|\psi_0\rangle + c_1|\psi_1\rangle$$

where $\hat{U}_H(0,t)$ is the propagator of $\hat{H}$, $|\psi_1\rangle$ is the state orthogonal to $|\psi_0\rangle$.

4. The quantum sensor is transformed into a superposition of observable readouts: $|\alpha\rangle = \hat{U}_b|\psi(t)\rangle = c_0'|0'\rangle + c_1'|1'\rangle$.

5. The final state of the quantum sensor is read out. The readout is assumed to be projective, a Bernoulli process that yields "0" with probability $1-p'$ and "1" with probability p' where $p' = |c_1'|^2 \propto p = 1 - |c_0|^2$ i.e. prop to transition probability of qubit changing its state during t.

⑥ Steps 1-5 represent a single measurement cycle. But this needs to be repeated and averaged over a large nr. of cycles N to estimate p. The repetition can be done sequentially on the same system or in parallel by averaging over an ensemble of N identical systems. (Non-interacting of course?)

⑦ In many situations we need to record a set of values for p, not only one as in step 6. The transition probability p is measured as a function of time t or a parameter of H_c, and the desired signal V is inferred from the data record $\{p_i\}$ using a suitable procedure.