The control Hamiltonian $\hat{H}_{control}$

Its role is to control, tune or manipulate the quantum sensor in a prescribed way. In a sensing process we often need to manipulate the qubit before, during and after sensing. These manipulations are done using gate operations generated by $\hat{H}_{control}(t)$.

The control Hamiltonian produces a unitary evolution operator:

$$\hat{U}_{control} = T \exp\left[-\frac{i}{\hbar}\int \hat{H}_{control}(t)\, dt\right]$$

where T denotes time ordering. For a short, well-designed pulse, this unitary becomes a quantum gate. There are several common gate operations. The Hadamard gate is used to create superpositions, e.g., $H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$.

A Hadamard-like operation can prepare the sensor in a state that is sensitive to phase accumulation. Another gate operation can be the Pauli-X and Pauli-Y gates: $\hat{\sigma}_x$, $\hat{\sigma}_y$ who corresponds to rotations in the qubit state about the x- and y-axes.

A general rotation is $R_x(\theta) = e^{-i\theta\hat{\sigma}_x/2}$ and $R_y(\theta) = e^{-i\theta\hat{\sigma}_y/2}$. Typically π-pulse flips the qubit, while $\pi/2$-pulse creates or analyzes a superposition.

If the sensor involves more than one qubit, then conditional gates may be required. Examples are CNOT gate and controlled phase gate. These gates connect the states of two qubits. For example, a CNOT applies a bit flip to one qubit depending on the state of another qubit. So for multi-qubit sensing, control Hamiltonians can generate entangling gates.

The control Hamiltonian can systematically tune the transition frequency ω_0 which means that the control field can modify the qubit energy splitting $\hbar\omega_0$. This capability is frequently exploited in noise spectroscopy experiments. The lesson here is that the control Hamiltonian can be used for gates AND tuning what frequencies the sensor is sensitive to.