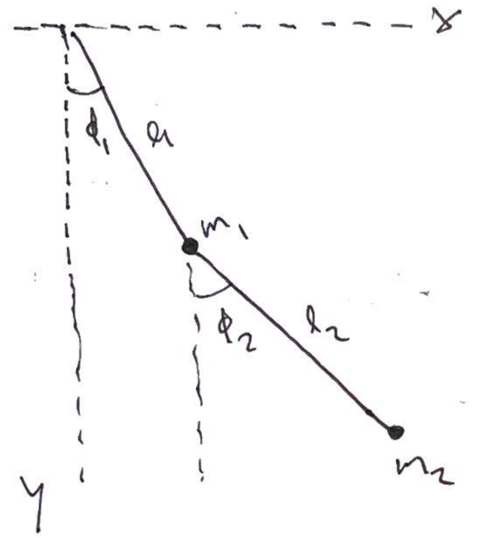
"Find the Lagrangian for each of the following systems when placed in a uniform gravitational field"



Take ϕ_1 and ϕ_2 as generalized coordinates. We will solve for m_1 first then m_2 and add them together.

Let m_1 have coordinates (x_1, y_1), then $T_1 = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2)$.

Since $x_1 = l_1 \sin \phi_1$ and $y = l_1 \cos \phi_1$ we get that $\dot{x}^2 + \dot{y}^2 = (\dot{\phi}_1 l_1 \cos \phi_1)^2 + (-\dot{\phi}_1 l_1 \sin \phi_1)^2 = l_1^2 \dot{\phi}_1^2$. For the potential we get $U_1 = -m_1 l_1 \cos \phi_1$ since y is chosen positive downward. In deriving the energy for m_2 we first note that given its position (x_2, y_2):

$$\begin{cases} x_2 = l_1 \sin \phi_1 + l_2 \sin \phi_2 \\ y_2 = l_1 \cos \phi_1 + l_2 \cos \phi_2 \end{cases} \rightsquigarrow \begin{cases} \dot{x}_2 = \dot{\phi}_1 l_1 \cos \phi_1 + \dot{\phi}_2 l_2 \cos \phi_2 \\ \dot{y}_2 = -\dot{\phi}_1 l_1 \sin \phi_1 - \dot{\phi}_2 l_2 \sin \phi_2 \end{cases}$$

With $\dot{x}_2^2 + \dot{y}_2^2 = (\dot{\phi}_1 l_1 \cos \phi_1 + \dot{\phi}_2 l_2 \cos \phi_2)^2 + (-\dot{\phi}_1 l_1 \sin \phi_1 - \dot{\phi}_2 l_2 \sin \phi_2)^2$

$$= \dot{\phi}_1^2 l_1^2 (\cos^2 \phi_1 + \sin^2 \phi_1) + \dot{\phi}_2^2 l_2^2 (\cos^2 \phi_2 + \sin^2 \phi_2) + \dots$$

$$+ 2 l_1 l_2 \dot{\phi}_1 \dot{\phi}_2 (\cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2)$$

where $\cos\phi_1 \cos\phi_2 + \sin\phi_1 \sin\phi_2 = \cos(\phi_1 - \phi_2)$

The kinetic energy of m_2 is therefore:

$$T_2 = \frac{1}{2} m_2 \left(l_1^2 \dot{\phi}_1^2 + l_2^2 \dot{\phi}_2^2 + 2 l_1 l_2 \dot{\phi}_1 \dot{\phi}_2 \cos(\phi_1 - \phi_2) \right)$$

and $U_2 = - m_2 g \left(l_1 \cos\phi_1 + l_2 \cos\phi_2 \right)$

The totals are given by $T = T_1 + T_2$ & $U = U_1 + U_2$:

$$T = \frac{1}{2} m_1 l_1^2 \dot{\phi}_1^2 + \frac{1}{2} m_2 \left(l_1^2 \dot{\phi}_1^2 + l_2^2 \dot{\phi}_2^2 + 2 l_1 l_2 \dot{\phi}_1 \dot{\phi}_2 \cos(\phi_1 - \phi_2) \right)$$

$$U = - m_1 g l_1 \cos\phi_1 - m_2 g \left(l_1 \cos\phi_1 + l_2 \cos\phi_2 \right)$$

So, the Lagrangian is

$$\boxed{\begin{aligned} \mathcal{L} = {} & \frac{1}{2}(m_1 + m_2) l_1^2 \dot{\phi}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\phi}_2^2 + m_2 l_1 l_2 \dot{\phi}_1 \dot{\phi}_2 \cos(\phi_1 - \phi_2) \\ & + (m_1 + m_2) g l_1 \cos\phi_1 + m_2 g l_2 \cos\phi_2 . \end{aligned}}$$