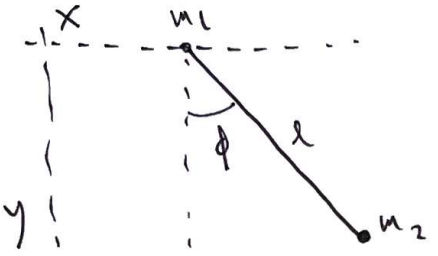
Problem 2.

"A simple pendulum of mass m_2, with mass m_1 at the point of support ..."



Take x and ϕ as generalized coordinates.

$T_1 = \frac{1}{2} m_1 \dot{x}^2$ & $U_1 = 0$

Let m_2 be located at (x_2, y_2):

$$x_2 = x + l \sin\phi \quad \rightsquigarrow \quad \dot{x}_2 = \dot{x} + l\dot{\phi}\cos\phi$$
$$y_2 = l\cos\phi \qquad\qquad \dot{y}_2 = -l\dot{\phi}\sin\phi$$

Thusly, $\dot{x}_2^2 = \dot{x}^2 + 2l\dot{x}\dot{\phi}\cos\phi + l^2\dot{\phi}^2\cos^2\phi$ and $\dot{y}_2^2 = l^2\dot{\phi}^2\sin^2\phi$

s.t., $T_2 = \frac{1}{2} m_2 \left[\dot{x}^2 + l^2\dot{\phi}^2 + 2l\dot{x}\dot{\phi}\cos\phi\right]$ and

$U_2 = -m_2 g l\cos\phi$. So and behold:

$$\mathcal{L} = \frac{1}{2}(m_1 + m_2)\dot{x}^2 + \frac{1}{2} m_2 (l^2\dot{\phi}^2 + 2l\dot{x}\dot{\phi}\cos\phi) + m_2 g l\cos\phi .$$