PROBLEM 2. "Find the ratio of the times in the same path for particles having ..."

Let's make use of the same scaling idea as in problem 1: $T \sim \frac{m\ell^2}{t^2}$. Supposing they follow the same trajectory, then $\mathcal{L}' = \lambda \mathcal{L}$ gives

$T' - U' = \lambda(T - U)$ where $U' = \alpha U$ since the Lagrangians must differ by an overall constant factor. So $T' - \alpha U = \lambda T - \lambda U$ means that $T' = \alpha T$. Using this fact and $T' \sim \frac{1}{t'^2}$ we get

$$\frac{1}{t'^2} = \alpha \frac{1}{t^2} \Rightarrow \frac{t'^2}{t^2} = \frac{1}{\alpha} \Rightarrow \boxed{\frac{t'}{t} = \sqrt{\frac{U}{U'}}}.$$