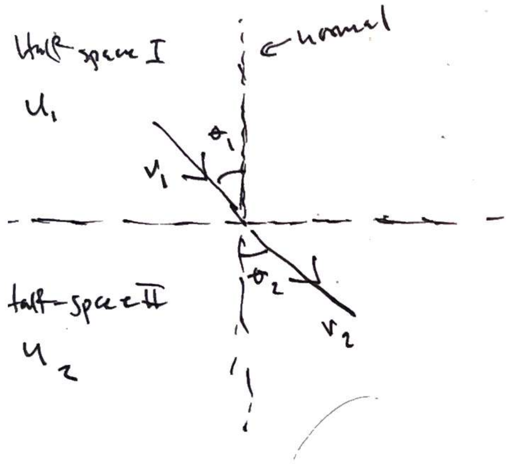
"A particle of mass m moving with velocity v_1 leaves half-space in which its potential energy ...."

The potentials U_1 and U_2 are constant which means that they do not depend on the coordinates. Consequently, the Lagrangian will not depend on the coordinates, leading to that the corresponding momentum components are conserved.



Weirdly looks like a classical analogue to Snell's law in optics. Specifically, the potential energy is independent of the coordinates whose axes are parallel to the plane (x and y) which is why we can claim that

$v_1 \sin\theta_1 = v_2 \sin\theta_2$. Furthermore, energy conservation says

$$\frac{1}{2}mv_1^2 + U_1 = \frac{1}{2}mv_2^2 + U_2 \iff v_2^2 = v_1^2 + 2[U_1 - U_2]/m.$$

Dividing the energy eq. with the momentum eq. gives us:

$$\frac{v_2^2}{(v_2 \sin\theta_2)^2} = \frac{v_1^2 + 2[U_1-U_2]/m}{(v_1 \sin\theta_1)^2} \Rightarrow \frac{\sin\theta_1}{\sin\theta_2} = \sqrt{1 + \frac{2[U_1-U_2]}{m v_1^2}}$$

(Btw, the same momentum conservation happens with Snell's law, which does not make it "weirdly" really.)