"obtain expressions for the Cartesian components and the magnitude of the angular momentum ..."

The angular momentum in cartesian coordinates are

$\bar{M} = \bar{r} \times \bar{p}$ where $r = (x, y, z)$ & $p = m(\dot{x}, \dot{y}, \dot{z})$.

$$\bar{r} \times \bar{p} = m \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x & y & z \\ \dot{x} & \dot{y} & \dot{z} \end{vmatrix} = m\left[\hat{x}(y\dot{z} - z\dot{y}) - \hat{y}(x\dot{z} - z\dot{x}) + \hat{z}(x\dot{y} - y\dot{x})\right]$$

Then, $M_x = m(y\dot{z} - z\dot{y})$, $M_y = m(z\dot{x} - x\dot{z})$, $M_z = m(x\dot{y} - y\dot{x})$

We have to express these in cylindrical coordinates:

$$\begin{cases} x = r\cos\phi \\ y = r\sin\phi \\ z = z \end{cases} \Rightarrow \begin{cases} \dot{x} = \dot{r}\cos\phi - r\dot{\phi}\sin\phi \\ \dot{y} = \dot{r}\sin\phi + r\dot{\phi}\cos\phi \\ \dot{z} = \dot{z} \end{cases}$$

$$M_x = m(r\sin\phi\,\dot{z} - z(\dot{r}\sin\phi + r\dot{\phi}\cos\phi)) = \underline{m(r\dot{z} - z\dot{r})\sin\phi - mrz\dot{\phi}\cos\phi}$$

$$M_y = m(z(\dot{r}\cos\phi - r\dot{\phi}\sin\phi) - r\cos\phi\,\dot{z}) = \underline{m(z\dot{r} - r\dot{z})\cos\phi - mrz\dot{\phi}\sin\phi}$$

$$M_z = m(r\cos\phi(\dot{r}\sin\phi + r\dot{\phi}\cos\phi) - r\sin\phi(\dot{r}\cos\phi - r\dot{\phi}\sin\phi)) = \underline{mr^2\dot{\phi}}$$

$$M^2 = M_x^2 + M_y^2 + M_z^2 = (m(r\dot{z} - z\dot{r})\sin\phi - mrz\dot{\phi}\cos\phi)^2 + (-m(r\dot{z} - z\dot{r})\cos\phi - mrz\dot{\phi}\sin\phi)^2$$
$$+ (mr^2\dot{\phi})^2 = m^2(r\dot{z} - z\dot{r})^2 + m^2r^2z^2\dot{\phi}^2 + m^2r^4\dot{\phi}^2$$

$$= \underline{m^2(r\dot{z} - z\dot{r})^2 + m^2r^2\dot{\phi}^2(r^2 + z^2)}.$$