Problem 1.

"Express the amplitude and initial phase of the oscillations in terms of the initial co-ordinate x_0 and velocity v_0."

The general solution to $\ddot{x} + \omega^2 x = 0$ is:

$$x(t) = a\cos(\omega t + \alpha)$$

This is the form we will use. Differentiate $x(t)$:
$\dot{x}(t) = -a\omega\sin(\omega t + \alpha)$. Evaluate $x(t)$ and $\dot{x}(t)$ when $t = 0$:

$$\begin{cases} x(0) = a\cos(\alpha) = x_0 & \text{(i)} \\ \dot{x}(0) = -a\omega\sin(\alpha) = v_0 \iff -a\sin(\alpha) = v_0/\omega & \text{(ii)} \end{cases}$$

Square (i) & (ii) and add them together:

$$a^2\cos^2(\alpha) + a^2\sin^2(\alpha) = x_0^2 + \frac{v_0^2}{\omega^2}$$

Hence $a = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$. To get the phase α, divide (ii) by (i):

$$-\frac{a\omega\sin(\alpha)}{a\cos(\alpha)} = \frac{v_0}{x_0}$$

$$\Rightarrow \tan\alpha = -\frac{v_0}{\omega x_0}.$$

(You could of course use the equivalent form of $x(t) = c_1\cos\omega t + c_2\sin\omega t$ instead.)