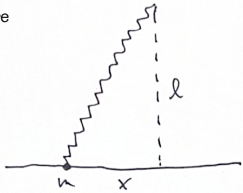
Problem 3

"Find the frequency of oscillations of a particle of mass m ....."

Figure



$\mathcal{L} = T - U$ where

$T = \frac{1}{2} m \dot{x}^2$ and the

potential $U(\delta l) \approx U(0) + \left.\frac{dU}{d(\delta l)}\right|_{\delta l = 0} \delta l \ldots\ldots$

$= F \delta l$ (where F is force of extension.)

The change in spring length δl is $\delta l = \sqrt{l^2 + x^2} - l$.

Binomial approx. tells us that $(1+\varepsilon)^n \approx 1 + n\varepsilon$ for small ε and this is proper when $\varepsilon \ll 1$. Applying this to δl: $\sqrt{l^2 + x^2} - l = l\sqrt{1 + \frac{x^2}{l^2}} - l$

$\approx l + \frac{x^2}{2l} - l = \frac{x^2}{2l}$. No need for explicit computation of EL-equations, instead recognize that since $U = \frac{F x^2}{2l}$ and $\omega = \sqrt{\frac{k}{m}}$ we get

$\boxed{\omega = \sqrt{\frac{F}{ml}}}$ given that $x \ll l$.