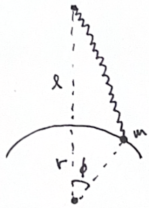
Problem 4

" The same as problem 3, but for a particle of mass m ..... "



Since the radius r is fixed, we note $T = \frac{1}{2} m r^2 \dot{\phi}$. Using the same idea as in problem 3

$$U(\delta l) \approx U(0) + \left.\frac{dU}{d(\delta l)}\right|_{\delta l = 0} \delta l \;.... = F \delta l$$

since F is the extension force. The spring length δl can be calculated with the cosine rule:

$$\delta l = \sqrt{r^2 + (r+l)^2 - 2r(r+l)\cos\phi} - l$$, for small ϕ, $\cos\phi = 1 - \frac{\phi^2}{2}$, leads to:

$$\delta l = \sqrt{r^2 + r^2 + 2rl + l^2 - 2r^2 - 2rl + r(r+l)\phi^2} - l$$

$$= l\sqrt{1 + r(r+l)\frac{\phi^2}{l^2}} - l$$ and for $\sqrt{1+\varepsilon} \approx 1 + \frac{\varepsilon}{2}$

when $\varepsilon << 1$ we can write $\delta l = l\left(1 + r(r+l)\frac{\phi^2}{2l^2}\right) - l$

$\iff \delta l = \frac{r(l+r)}{2l}\phi^2$. So, since $T = \frac{1}{2} m r^2 \dot{\phi}$ & $U = \frac{1}{2}\frac{Fr(l+r)}{l}\phi^2$

we have $\omega = \sqrt{\frac{k}{m}} = \boxed{\sqrt{\frac{F(l+r)}{mrl}}}$