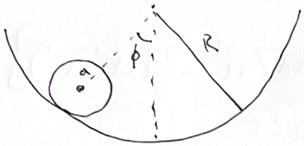
Problem 6. "Find the kinetic energy of a homogenuous cylinder ..."



Let M be the mass of the cylinder. Generally we have

$$T = \frac{1}{2} M v^2 + \frac{1}{2} I \Omega^2 .$$

The translational velocity $v = (R-a)\dot{\phi}$ and no-slipping condition $\Omega = \frac{v}{a} = \frac{(R-a)\dot{\phi}}{a}$. The moment of inertia along the cylinder axis is $I_3 = \int r^2 \, dm$ where $dm = \rho \, dV = \rho \, r \, dr \, d\theta \, dz$ for cylindrical coordinates.

$$I_3 = \rho \int_0^h dz \int_0^{2\pi} d\theta \int_0^a r^3 \, dr = \rho \cdot h \cdot 2\pi \cdot \frac{a^4}{4} .$$

Now, note that $\rho = \frac{M}{V} = \frac{M}{\pi a^2 \cdot h} \Rightarrow I_3 = \frac{M}{\pi a^2 h} \cdot h \cdot 2\pi \cdot \frac{a^4}{4} = \frac{1}{2} M a^2 .$

Hence, $T = \frac{1}{2} M (R-a)^2 \dot{\phi}^2 + \frac{1}{2} \cdot \frac{1}{2} M a^2 \cdot \left(\frac{(R-a)\dot{\phi}}{a} \right)^2$

$$\boxed{= \frac{3}{4} M (R-a)^2 \dot{\phi}^2}$$