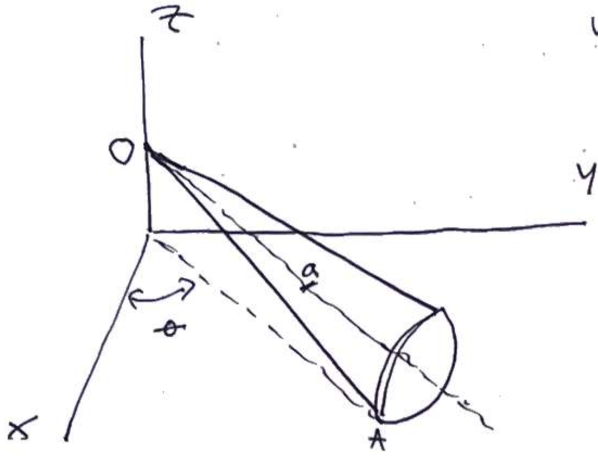
PROBLEM 8.

"Find the kinetic energy of a homogeneous cone whose base..."



This is identical to problem 7 but with an elevated vertex, and importantly, the cone axis is in the actual plane now. The general expression for the kinetic energy is $T = \frac{1}{2}\mu V^2 + \frac{1}{2} I \Omega^2$. Clearly $V = a\dot{\theta}$ and the rotation happens along OA, so $\Omega = \frac{V}{a \sin\alpha} = \frac{\dot{\theta}}{\sin\alpha}$.

If we think about the principal axes of the cone, we have that x_3 is along the cone axis and x_1, x_2 are perpendicular to the cone axis. Translated:

$$\Omega \sin\alpha = \frac{\dot{\theta}}{\sin\alpha} \sin\alpha = \dot{\theta} \quad (\text{for } \perp \text{ cone axis})$$

$$\Omega \cos\alpha = \frac{\dot{\theta}}{\sin\alpha} \cos\alpha = \dot{\theta} \cot\alpha \quad (\text{for } \parallel \text{ cone axis})$$

That is, $\vec{\Omega} = (\dot{\theta}, 0, \dot{\theta}\cot\alpha)$. More explicitly, the kinetic energy is $T = \frac{1}{2}\mu a^2 \dot{\theta}^2 + \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2} I_3 \dot{\theta}^2 \cot^2\alpha$.

From problem 2(e) or 7, we already made it clear that $a = \frac{3}{4}h$, $I_1 = \frac{3}{20}\mu\left(R^2 + \frac{h^2}{4}\right)$, $I_3 = \frac{3}{10}\mu R^2$.

$$T = \frac{1}{2}\mu\left(\frac{3h}{4}\right)^2\dot{\theta}^2 + \frac{1}{2}\cdot\frac{3}{20}\mu\left(R^2 + \frac{h^2}{4}\right)\dot{\theta}^2 + \frac{1}{2}\frac{3}{10}\mu R^2\dot{\theta}^2\cot^2\alpha$$

using $R = h\tan\alpha$: $T =$

$$\frac{9}{32}\mu h^2\dot{\theta}^2 + \frac{3}{40}\mu\left(h^2\tan^2\alpha + \frac{h^2}{4}\right)\dot{\theta}^2 + \frac{3}{20}\mu h^2\tan^2\alpha\,\dot{\theta}^2\cot^2\alpha$$

$$= \mu h^2\dot{\theta}^2\left[\frac{9}{32} + \frac{3}{160} + \frac{3}{20} + \frac{3}{40}\tan^2\alpha\right]$$

$$= \mu h^2\dot{\theta}^2\left[\frac{45}{160} + \frac{3}{160} + \frac{24}{160} + \frac{3}{40}\tan^2\alpha\right]$$

$$= \mu h^2\dot{\theta}^2\left[\frac{72}{160} + \frac{3}{40}\tan^2\alpha\right] = \frac{3\mu h^2\dot{\theta}^2}{40}\left[6 + \tan^2\alpha\right]$$

$$\boxed{= \frac{3\mu h^2\dot{\theta}^2}{40}\left[5 + \sec^2\alpha\right]}$$

The lesson from Problem 7 & 8 is really to get a handle on the projection, decompose $\vec{\Omega}$ and connect the inertia tensor property.