Problem 2

The Lagrangian for a uniformly rotating frame:

$$\mathcal{L} = \frac{1}{2}mv^2 + m\bar{v}\cdot\bar{\Omega}\times\bar{r} + \frac{1}{2}m(\bar{\Omega}\times\bar{r})^2 - U.$$

The procedure is the same, find $\bar{p}$, invert to get $\bar{v}$ in terms of $\bar{p}$, substitute into $H = \bar{p}\cdot\bar{v} - \mathcal{L}$.

The momentum is $\bar{p} = \partial\mathcal{L}/\partial\bar{v} = mv + m\bar{\Omega}\times\bar{r}$. which is different from the standard case since we have an added coriolis term. Invert: $\bar{v} = \frac{\bar{p}}{m} - \bar{\Omega}\times\bar{r}$.

substituting $\bar{v}$ in the Hamiltonian:

$$H = \bar{p}\cdot\left(\frac{\bar{p}}{m} - \bar{\Omega}\times\bar{r}\right) - \frac{1}{2}m\left(\frac{\bar{p}}{m} - \bar{\Omega}\times\bar{r}\right)^2 - m\left(\frac{\bar{p}}{m} - \bar{\Omega}\times\bar{r}\right)\cdot\bar{\Omega}\times\bar{r} + \dots$$

$$- \frac{1}{2}m(\bar{\Omega}\times\bar{r})^2 + U$$

$$= \frac{|\bar{p}|^2}{m} - \bar{p}\cdot(\bar{\Omega}\times\bar{r}) - \frac{|\bar{p}^2|}{2m} + \bar{p}\cdot(\bar{\Omega}\times\bar{r}) - \frac{1}{2}m(\bar{\Omega}\times\bar{r})^2 - \bar{p}\cdot(\bar{\Omega}\times\bar{r}) + \dots$$

$$+ m(\bar{\Omega}\times\bar{r})^2 + U - \frac{1}{2}m(\bar{\Omega}\times\bar{r})^2$$

$$\boxed{= \frac{|\bar{p}|^2}{2m} - \bar{p}\cdot(\bar{\Omega}\times\bar{r}) + U}$$