Problem 7

"Determine the Poisson brackets formed from the Cartesian components of momentum $\bar{p}$ and the angular momentum $\bar{M} = \bar{r} \times \bar{p}$ of a particle."

$$\bar{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix} = \hat{i}(y p_z - z p_y) - \hat{j}(x p_z - z p_x) + \hat{k}(x p_y - y p_x)$$

$$M_x = y p_z - z p_y, \quad M_y = z p_x - x p_z, \quad M_z = x p_y - y p_x$$

Using the fact that $[f, p_k] = -\partial f / \partial q_k$:

$[M_x, p_x] = 0$, $[M_x, p_y] = -\partial M_x / \partial y = -p_z$ and

$[M_x, p_z] = -\partial M_x / \partial z = p_y$. The remaining brackets follow by cyclically permuting $x \mapsto y \mapsto z \mapsto x$:

$[M_y, p_y] = 0$, $[M_y, p_z] = -\partial M_y / \partial z = -p_x$ and

$[M_y, p_x] = -\partial M_y / \partial x = p_z$. Now let's do the cyclic permutation of M_y:

$[M_z, p_z] = 0$, $[M_z, p_x] = -\partial M_z / \partial x = -p_y$ and

$[M_z, p_y] = -\partial M_z / \partial y = p_x$.