Problem 2.

"Determine the Poisson brackets formed from the components of $\overline{M}$."

The components are $M_x = yP_z - zP_y$, $M_y = zP_x - xP_z$ and $M_z = xP_y - yP_x$. using the definition:

$$[M_x, M_y] = \sum_u \left(\frac{\partial M_x}{\partial P_u} \frac{\partial M_y}{\partial q_u} - \frac{\partial M_x}{\partial q_u} \frac{\partial M_y}{\partial P_u} \right)$$

$$= \frac{\partial M_x}{\partial P_x} \frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial x} \frac{\partial M_y}{\partial P_x} + \frac{\partial M_x}{\partial P_y} \frac{\partial M_y}{\partial y} - \frac{\partial M_x}{\partial y} \frac{\partial M_y}{\partial P_y}$$

$$+ \frac{\partial M_x}{\partial P_z} \frac{\partial M_y}{\partial z} - \frac{\partial M_x}{\partial z} \frac{\partial M_y}{\partial P_z}$$

$$= yP_x - (-P_y)(-x) = yP_x - xP_y$$

$$= -(xP_y - yP_x) = -M_z$$

By cyclic permutation $x \mapsto y \mapsto z \mapsto x$. starting from $[M_x, M_y] = -M_z$ we get:

$[M_y, M_z] = -M_x$ and apply once more

$[M_z, M_x] = -M_y$

Hence, $\boxed{[M_x, M_y] = -M_z, \ [M_y, M_z] = -M_x, \ [M_z, M_x] = -M_y.}$