"Show that the collisionless dissipation Q is always positive in an isotropic plasma."

The energy dissipated per unit time and volume by a longitudinal wave in the plasma is:

$$Q = -|E|^2 \frac{\pi m e^2 \omega}{2k^2} \left[\frac{df(P_x)}{dP_x}\right]_{P_x = \omega/k}$$

showing that $Q > 0$ requires $df(P_x)/dP_x < 0$.
In thinking about $f(P_x)$, let's set up the $\vec{k}$ along the x-axis. That is, any momentum $\bar{p}$ splits into P_x along the wave and $P_\perp^2 = P_y^2 + P_z^2$ perp. to it. We only care about electrons moving along the wave since otherwise it won't exchange energy with it. To get the 1D reduced distribution we integrate out the transverse stuff:

$$f(P_x) = \int f(\bar{p})\, dP_y\, dP_z$$

We are told that f is isotropic, i.e., it only depends on $|\bar{p}|^2$. To make use of this symmetry we can convert to polar coordinates in the $P_y - P_z$ plane. The area element becomes $dP_y dP_z = 2\pi P_\perp dP_\perp$:

$$f(P_x) = \int_0^\infty f(P_x^2 + P_\perp^2)\, 2\pi P_\perp\, dP_\perp$$

where we can write $2\pi P_\perp dP_\perp$ in terms of $d(P_\perp^2)$ s.t. $2\pi P_\perp dP_\perp = \pi d(P_\perp^2)$ since $d(P_\perp^2) = 2P_\perp dP_\perp$.

Therefore we can write:

$$f(p_x) = \int_0^\infty f(p_x^2 + p_\perp^2)\, \pi\, d(p_\perp^2)$$

Differentiate w.r.t. p_x:

$$\frac{d f(p_x)}{d p_x} = 2\pi p_x \int_0^\infty f'(p_x^2 + p_\perp^2)\, d(p_\perp^2)$$

where $f' = df / d(p^2)$ so by the fundamental theorem of calculus:

$$\int_0^\infty \frac{d}{d(p_\perp^2)} f(p_x^2 + p_\perp^2)\, d(p_\perp^2) = \left[f(p_x^2 + p_\perp^2) \right]_0^\infty$$

$$= f(\infty) - f(p_x^2)$$ and since $f(\infty) = 0$ we

have that $\dfrac{d f(p_x)}{d p_x} = -2\pi p_x f(p_x^2)$.

And $p_x =$ [illegible] with $f(p_x^2)$ non-negative means

that $\boxed{Q \geq 0}$.

$\square$