Problem

A charged particle in an EM-field is subject to a force $\frac{d\bar{p}}{dt} = e\bar{E} + \frac{e}{c}\bar{v} \times \bar{H}$. The relativistic energy of the particle is $\mathcal{E}_{kin} = \gamma m c^2$ and $\bar{p} = \gamma m \bar{v}$, from this we can express $\bar{p}$ as $\bar{p} = \frac{\mathcal{E}_{kin}}{c^2}\bar{v}$. (Through this we can make use of $\frac{d\mathcal{E}_{kin}}{dt} = e\bar{E}\cdot\bar{v}$.) Substituting $\bar{p} = \frac{\mathcal{E}_{kin}}{c^2}\bar{v}$ in the LHS of the Lorentz force eq:

$$\frac{d\bar{p}}{dt} = \frac{d}{dt}\left(\frac{\mathcal{E}_{kin}}{c^2}\bar{v}\right) = \frac{1}{c^2}\left(e(\bar{E}\cdot\bar{v})\bar{v} + \mathcal{E}_{kin}\dot{\bar{v}}\right).$$

Equating this with the RHS:

$$\frac{1}{c^2}\left(e(\bar{E}\cdot\bar{v})\bar{v} + \mathcal{E}_{kin}\dot{\bar{v}}\right) = e\bar{E} + \frac{e}{c}\bar{v}\times\bar{H}$$

$$\Leftrightarrow \quad \dot{\bar{v}} = \frac{ec^2}{\mathcal{E}_{kin}}\left[\bar{E} + \frac{1}{c}\bar{v}\times\bar{H} - \frac{1}{c^2}\bar{v}(\bar{v}\cdot E)\right]$$

and since $\mathcal{E}_{kin} = \gamma m c^2 \Rightarrow \dot{\bar{v}} = \frac{ec^2}{\gamma m c^2}\left[\bar{E} + \frac{1}{c}\bar{v}\times\bar{H} - \frac{1}{c^2}\bar{v}(\bar{v}\cdot\bar{E})\right]$

$$= \frac{e}{m}\sqrt{1-\frac{v^2}{c^2}}\left(\bar{E} + \frac{1}{c}\bar{v}\times\bar{H} - \frac{1}{c^2}\bar{v}(\bar{v}\cdot\bar{E})\right)$$

$\square$

(By doing this problem we realize that relativistic dynamics is given by $d\bar{p}/dt$ not $m\bar{a}$ and it is really the energy-momentum that determines the acceleration.)