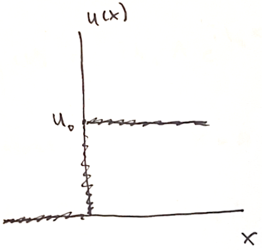
PROBLEM 1.

"Determine the reflection coefficient of a particle from a rectangular potential wall ..."



TISE is $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + u(x)\psi(x) = E\psi(x)$

$$\Rightarrow \frac{d^2}{dx^2}\psi(x) = -\frac{2m}{\hbar^2}\left[E - u(x)\right]\psi(x)$$

For $x < 0$ we have a free particle $\psi_I(x) = e^{ik_1 x} + Be^{-ik_1 x}$ where we have chosen the coefficient in front of $e^{ik_1 x}$ to be 1 according to convention. For $x > 0$ we only have a transmitted wave $\psi_{II}(x) = Ae^{ik_2 x}$. Note that $k_1 = \frac{\sqrt{2mE}}{\hbar}$ & $k_2 = \frac{\sqrt{2m[E-u_0]}}{\hbar}$. The boundary conditions require ψ and ψ' to be continuous at $x=0$. In other words, the value of the wavefunction just to the left $\psi(0^-)$ must equal the value just to the right $\psi(0^+)$. Since $\psi_I(0) = 1 + B$ for $x < 0$ and $\psi_{II}(0) = A$ for $x > 0$ we get that

$1 + B = A$.

PROBLEM II

Applying the continuity condition on ψ':

$$\psi'_I(x) = ik_1 e^{ik_1 x} - ik_1 B e^{-ik_1 x}$$

$$\psi'_{II}(x) = ik_2 A e^{ik_2 x}$$

Evaluate at $x = 0 \Rightarrow \psi'_I(0) = ik_1(1-B)$

and $\psi'_{II}(0) = ik_2 A$ gives $k_1(1-B) = k_2 A$.

With the two equations $1+B = A$ and $k_1(1-B) = k_2 A$

we get $A = \dfrac{2k_1}{k_1+k_2}$ and $B = \dfrac{k_1-k_2}{k_1+k_2}$.

The reflection coefficient R is defined $R = |B|^2$,

so $\boxed{R = \left|\dfrac{k_1-k_2}{k_1+k_2}\right|^2}$. Now since $p = \hbar k$ you

could also answer $R = \left|\dfrac{\frac{p_1}{\hbar} - \frac{p_2}{\hbar}}{\frac{p_1}{\hbar} + \frac{p_2}{\hbar}}\right|^2 \boxed{= \left|\dfrac{p_1-p_2}{p_1+p_2}\right|^2}$

which is clearer since it shows that reflection happens because the particle's momentum changes across the boundary.