Problem 2.

"Find the mean square fluctuation of the velocity."

The quantity we want to compute is $\overline{(\Delta V)^2}$ where $\Delta V = v - \bar{v}$ measures how much the speed deviates from its mean, which makes $\overline{(\Delta V)^2}$ the variance.

We have $\overline{(\Delta V)^2} = \overline{(V - \bar{V})^2} = \overline{V^2 - 2V\bar{V} + \bar{V}^2}$.

Consider that $\bar{V}$ is just a number which means that $\overline{2v\bar{v}} = 2\bar{v}\bar{v} = 2\bar{v}^2$ s.t. $\overline{v^2} - 2\bar{v}^2 + \bar{v}^2 = \overline{V^2} - \bar{V}^2$.

$\overline{V^2}$ is obtained from problem 1 where we set $n=2$, i.e., $r=1$. Using the even case result: $\overline{v^{2r}} = \left(\frac{T}{m}\right)^r (2r+1)!!$

we get $\overline{V^2} = \frac{T}{m} \cdot 3!! = \frac{3T}{m}$. And similarly, $\bar{v}^2$ is obtained from problem 1 where we use the odd case result for $n=1$, i.e., $r=0$ for $\overline{v^{2r+1}} = \frac{2}{\sqrt{\pi}}\left(\frac{2T}{m}\right)^{(2r+1)/2}(r+1)!$

so $\bar{V} = \frac{2}{\sqrt{\pi}}\sqrt{\frac{2T}{m}} \Rightarrow \bar{V}^2 = \frac{8T}{\pi m}$ which gives us:

$$\overline{(\Delta V)^2} = \overline{V^2} - \bar{V}^2 = \frac{3T}{m} - \frac{8T}{\pi m} = \boxed{\frac{T}{m}\left(3 - \frac{8}{\pi}\right)}.$$

Since $3 > \frac{8}{\pi}$ the spread of the speed distribution grows with temperature T around the mean and decreases with heavier particles.