

Problem 3

"Find the mean energy, the mean square energy and the mean square fluctuation of the kinetic energy of an atom."

So, we are looking for the mean energy $\bar{\epsilon}$, the mean square energy $\overline{\epsilon^2}$ and then $\overline{(\Delta\epsilon)^2}$ for fluctuations.

All of these can be obtained from problem 1.

For the mean energy $\bar{\epsilon} = \frac{1}{2} m \overline{v^2}$ we computed $\overline{v^2}$ in problem 2 with $n=2, r=1 \Rightarrow \bar{\epsilon} = \frac{1}{2} m \frac{3T}{m} = \frac{3T}{2}$. The mean square energy $\overline{\epsilon^2}$, we need $\overline{v^4}$ since $\overline{\epsilon^2} = \frac{1}{4} m^2 \overline{v^4}$, and this is $\overline{v^4} = \left(\frac{T}{m}\right)^2 (2-2+1)!! = \frac{T^2}{m^2} 5!! = \frac{15T^2}{m^2}$ which yields:

$$\overline{\epsilon^2} = \frac{1}{4} m^2 \overline{v^4} = \frac{1}{4} m^2 \cdot \frac{15T^2}{m^2} = \frac{15T^2}{4} \quad \text{For the fluctuations } \overline{(\Delta\epsilon)^2}:$$

$$\overline{(\Delta\epsilon)^2} = \overline{\epsilon^2} - \bar{\epsilon}^2 = \frac{15T^2}{4} - \left(\frac{3T}{2}\right)^2 = \frac{3T^2}{2}$$

(These results follow from problem 1 just by plugging in the correct "n".)