Problem 4.

"Find the probability distribution for the kinetic energy of an atom."

The distribution in terms of v is:

$dw_v = 4\pi \left(\frac{m}{2\pi T}\right)^{3/2} e^{-mv^2/2T} v^2 \, dv$. Now, $\varepsilon = \frac{1}{2} m v^2$

so $v = \sqrt{\frac{2\varepsilon}{m}}$. The differential $d\varepsilon = m v \, dv$ becomes

$$dv = \frac{d\varepsilon}{mv} = \frac{d\varepsilon \sqrt{m}}{m\sqrt{2\varepsilon}} = \frac{d\varepsilon}{\sqrt{2m\varepsilon}}.$$

Let's substitute these into dw_v:

$$dw_v = 4\pi \left(\frac{m}{2\pi T}\right)^{3/2} e^{-m\frac{2\varepsilon}{2mT}} \frac{2\varepsilon}{m} \cdot \frac{d\varepsilon}{\sqrt{2m\varepsilon}}$$

$$= \frac{4\pi m^{3/2}}{2^{3/2}\pi^{3/2} T^{3/2}} \cdot \frac{\sqrt{2\varepsilon}}{m^{3/2}} e^{-\frac{\varepsilon}{T}} \, d\varepsilon$$

$$= \boxed{\frac{2}{\sqrt{\pi T^3}} e^{-\frac{\varepsilon}{T}} \sqrt{\varepsilon} \, d\varepsilon}$$