Problem 1. "Find the density of a gas in a cylinder of radius R and length l rotating ...."

By "density" we mean the number density which is given by $n(\bar{r}) = n_0 e^{-u(\bar{r})/T}$ where $u(\bar{r})$ is the potential energy. From earlier chapters we are told that the rotation of a body is equivalent to the presence of an external field. Basically, the molecules are subject to a centrifugal force $F = m\Omega^2 r \Rightarrow -\frac{du}{dr} = m\Omega^2 r \Rightarrow u = -\frac{1}{2}m\Omega^2 r^2$. The density is therefore $\boxed{n(r) = n_0 e^{m\Omega^2 r^2/2T}}$. To get the normalization constant n_0 we make use of $dN_r = n_0 e^{-u(\bar{r})/T} dV$ where dN_r is the nr of molecules in the volume element dV:

$$N = \int_0^R n_0 e^{m\Omega^2 r^2/2T} \cdot 2\pi r l \, dr = \left[\begin{aligned} u &= \frac{m\Omega^2 r^2}{2T} \\ du &= \frac{m\Omega^2}{T} r \, dr \end{aligned} \right]$$

$$= 2\pi l n_0 \frac{T}{m\Omega^2} \int_0^{m\Omega^2 R^2/2T} e^u \, du = 2\pi l n_0 \frac{T}{m\Omega^2} \left(e^{m\Omega^2 R^2/2T} - 1 \right)$$

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Solving for n_0:

$$n_0 = \frac{N m \Omega^2}{2\pi \ell T \left[e^{m\Omega^2 R^2 / 2T} - 1 \right]}$$

hence the normalized density is

$$\boxed{n(r) = \frac{N m \Omega^2 e^{m\Omega^2 r^2 / 2T}}{2\pi \ell T \left[e^{m\Omega^2 R^2 / 2T} - 1 \right]}}$$

(What is nice about this problem is how the rotation simply becomes a potential problem.)