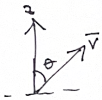
PROBLEM I

"Find the number of impacts of gas molecules on unit area of the wall per unit time ....."

Suppose we have some container with molecules hitting the walls. Take a snapshot of the wall:

with sidelength 1. Then let the z-axis be $\odot$.

Clearly only $V_z > 0$ hit the wall. Let the velocity $\bar{V}$ make an angle θ with z-axis:



s.t. $V_z = V \cos\theta$.

The nr of collisions of molecules per unit time & area of the wall surface is given by:

$$d\nu_v = \frac{N}{V}\left(\frac{m}{2\pi T}\right)^{3/2} \exp\left[-m(v_x^2 + v_y^2 + v_z^2)/2T\right] \times v_z \, dv_x \, dv_y \, dv_z$$

This can be written in the velocity space where $dv_x \, dv_y \, dv_z = v^2 \sin\theta \, dv \, d\theta \, d\phi$. Hence, we get

$$d\nu_v = \frac{N}{V}\left(\frac{m}{2\pi T}\right)^{3/2} e^{-mv^2/2T} v^3 \sin\theta \cos\theta \, dv \, d\theta \, d\phi.$$

As requested, we are interested in the nr of impacts between θ and $\theta + d\theta$, translated: integrate all speeds v from 0 to ∞ and ϕ from 0 to 2π.

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$$d\nu_\theta = \frac{N}{V}\left(\frac{m}{2\pi T}\right)^{3/2} \sin\theta \cos\theta \, d\theta \int_0^{2\pi} d\phi \int_0^{\infty} e^{-mv^2/2T} v^3 \, dv$$

Let $I = \int_0^\infty e^{-mv^2/2T} v^3 \, dv = \left[\begin{array}{l} u = \frac{m}{2T} v^2 \\ du = \frac{m}{T} v \, dv \end{array}\right]$

$$= \frac{2T^2}{m^2} \int_0^\infty u e^{-u} \, du = \frac{2T^2}{m^2}$$

So $d\nu_\theta = \frac{N}{V}\left(\frac{m}{2\pi T}\right)^{3/2} \sin\theta \cos\theta \, d\theta \; 2\pi \cdot \frac{2T^2}{m^2}$

$$\boxed{= \frac{N}{V}\left(\frac{2T}{m\pi}\right)^{1/2} \sin\theta \cos\theta \, d\theta}$$

What we notice here is that surfaces only see a version of the velocity distribution despite the fact that velocities in the gas are isotropic.