Problem 7.

"Find the work done on an ideal gas in an isothermal change of volume from V_1 to V_2."

The work done is the change in free energy F. The reason is that since $dE = TdS - PdV$ and $dF = dE - TdS - SdT$ we can show that:

$dF = TdS - PdV - TdS - SdT = -PdV - SdT$. And $dT = 0$ because the process is isothermal, so $dF = -PdV$. The assumption of ideal gas means we can write $P = NT/V$, hence:

$$R = F_2 - F_1 = -\int_{V_1}^{V_2} P\,dV = -\int_{V_1}^{V_2} \frac{NT}{V}\,dV = NT\log\left(\frac{V_1}{V_2}\right)$$

If you would like, we could also write $R = NT\log\left(\frac{P_2}{P_1}\right)$. The heat absorbed $Q = T\Delta S = T(S_2 - S_1)$ where the entropy $S = -\partial F/\partial T$. But simpler is to use the fact that $\Delta E = 0$ isothermally, i.e., $Q = -R = -NT\log(V_2/V_1) = NT\log(V_2/V_1)$.