Problem 4.

The change in entropy is $\Delta S = S_f - S_i$, which means that we have to find espressions for the initial and final entropy.

The entropy of an ideal gas of N particles whose heat capacity is constant is $S = Nc_p \log T - N \log P$. Thus we need to find the final temperature T and final pressure P.

The energy of an ideal gas is $E = Nc_v T$, the principle of energy conservation says:

$$N c_v T_1 + N c_v T_2 = 2 N c_v T$$

Hence the final temp is the average $T = \frac{T_1 + T_2}{2}$.

To find P_{final}, consider that after connecting the vessels we have $P_{final} = \frac{(2N)\,T}{V_1 + V_2} = \frac{2N \cdot \frac{T_1 + T_2}{\cancel{2}}}{\frac{N(T_1 + T_2)}{P}} = P$,

i.e., pressure is unchanged.

The initial entropy is $S_i = (Nc_p \log T_1 - N \log P) + (Nc_p \log T_2 - N \log P) = -2N \log P + Nc_p \log (T_1 T_2)$.

The final entropy is $S_f = 2Nc_p \log\left(\frac{T_1 + T_2}{2}\right) - 2N \log P$.

So, the change in entropy is $\Delta S = S_f - S_i = Nc_p \log \frac{(T_1 + T_2)^2}{4 T_1 T_2}$.