Problem 2.

Let R denote the work done on the gas where $R > 0$ means work is done on the gas (compressing). Adiabatic means no heat exchange with the environment, $Q = 0$. The first law of Thermo says that $\Delta E = Q + R$, thus $R = E_2 - E_1$. With the energy of an ideal gas this becomes $R = Nc_v(T_2 - T_1)$ since $E_i = Nc_v T_i$.

This is a complete answer in terms of temperatures, but often we know the volumes, not final temperatures. Consider the entropy as a function of (T, V):

$$S = Nc_v \log T + N \log V + \text{const}$$

and set $dS = 0$ s.t. $c_v \frac{dT}{T} + \frac{dV}{V} = 0$. Integrating this eq. gives $c_v \log T + \log V = \text{const}$, and taking the exponential we get $T^{c_v} V = \text{const} \Rightarrow T V^{1/c_v} = \text{const}$.

Since $c_p - c_v = 1$, then $\frac{1}{c_v} = \frac{c_p - c_v}{c_v} = \frac{c_p}{c_v} - 1 = \gamma - 1$

where $\gamma = c_p / c_v$, hence $T V^{\gamma - 1} = \text{const}$. Applying this to the initial and final states: $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ gives

$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$. Substituting T_2 into the eq. for R:

$$R = Nc_v T_1 \left[\left(\frac{V_1}{V_2}\right)^{\gamma-1} - 1\right] = Nc_v T_2 \left[1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}\right]$$

(same expression using T_1 or T_2)