Problem 5.

A polytropic process follows $PV^n = a = \text{const}$, where n is a fixed number. Different values of n describe different processes.

Work done on the gas is $R = -\int_{V_1}^{V_2} P\,dV = -\int_{V_1}^{V_2} aV^{-n}\,dV$

$$= -a\,\frac{V_2^{1-n} - V_1^{1-n}}{1-n} = \frac{a}{n-1}\left(V_2^{1-n} - V_1^{1-n}\right)$$

using the equation of state $aV^{1-n} = NT$ at each point:

$$\frac{a}{n-1}\left(V_2^{1-n} - V_1^{1-n}\right) = \frac{N(T_2 - T_1)}{n-1}$$

The first law of Thermo says $\Delta E = Q + R \Rightarrow Q = \Delta E - R$.
With $\Delta E = NC_v(T_2 - T_1)$:

$$Q = NC_v(T_2 - T_1) - \frac{N(T_2 - T_1)}{n-1} = N(T_2 - T_1)\left(C_v - \frac{1}{n-1}\right)$$

using $C_v + 1 = C_p$ & $C_p/C_v = \gamma$:

$$Q = NC_v(T_2 - T_1)\left(\frac{n-\gamma}{n-1}\right)$$

(This problem is the capstone for problems 2–5, e.g., $n=0$ gives isobaric (Problem 4), $n=1$ gives isothermal and $n=\gamma$ is adiabatic.)