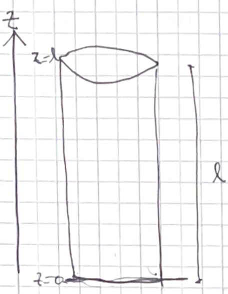
In §2 Landau introduces the stress tensor σ_{ik} representing the internal forces in the material. Note that $i,k = 1,2,3 \leftrightarrow x,y,z$.



Deformation is described by the quants u_x, u_y & u_z. The equilibrium eq. $\partial_k \sigma_{ik} + \rho g_i = 0$ determines σ_{ik}. The symmetry of the rod suggests compression in z, which means that only σ_{zz} is non-zero. Choosing gravity ↓ then $\partial \sigma_{zz} / \partial_z = \rho g$. Integrating this with B.C. $z = l$, $\sigma_{zz} = -\rho g (l - z)$. $[\sigma_{zz}(l) = 0]$

from eq 5.14 $\sigma_{ik} = \frac{E}{1+\sigma}\left(u_{ik} + \frac{\sigma}{1-2\sigma} u_{ll} \delta_{ik}\right)$

for xx: $0 = \frac{E}{1+\sigma}\left(u_{xx} + \frac{\sigma}{1-2\sigma}(u_{xx} + u_{yy} + u_{zz})\right)$

for yy: $0 = \frac{E}{1+\sigma}\left(u_{yy} + \frac{\sigma}{1-2\sigma}(u_{xx} + u_{yy} + u_{zz})\right)$

i.e., $u_{xx} = u_{yy} = -\sigma u_{zz}$. And for zz:

$$\sigma_{zz} = \frac{E}{1+\sigma}\left(u_{zz} + \frac{\sigma}{1-2\sigma}(u_{xx} + u_{yy} + u_{zz})\right)$$

$\Rightarrow u_{zz} = \frac{\sigma_{zz}}{E}$, and $u_{xx} = u_{yy} = -\sigma \frac{\sigma_{zz}}{E}$.

From the equilibrium condition we found

$\sigma_{zz} = -\rho g(l-z)$ which gives the strains:

$$u_{zz} = -\frac{\rho g(l-z)}{E}, \quad u_{xx} = u_{yy} = \frac{\sigma \rho g(l-z)}{E}.$$

Now, since $u_{xx} = \partial u_x/\partial x$, $u_{yy} = \partial u_y/\partial y$, $u_{zz} = \partial u_z/\partial z$

we get $u_x = \sigma\rho g(l-z)x/E$, $u_y = \sigma\rho g(l-z)y/E$

and for $u_z = -\frac{\rho g}{E}\left(lz - \frac{z^2}{2}\right) + f(x,y)$. To find

$f(x,y)$ we need to make use of $u_{xz} = u_{yz} = 0$.

The mixed strains $u_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) = 0$

and $u_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) = 0$. Using the

relations for u_x & u_y above these must yield

$$\frac{\partial u_x}{\partial z} = -\frac{\sigma\rho g}{E}x, \quad \frac{\partial u_y}{\partial z} = -\frac{\sigma\rho g}{E}y \quad \text{so}$$

$$\frac{\partial u_z}{\partial x} = \frac{\sigma\rho g}{E}x \quad \text{and} \quad \frac{\partial u_z}{\partial y} = \frac{\sigma\rho g}{E}y. \text{ By}$$

integrating these two we get $f(x,y) = \frac{\sigma\rho g}{2E}(x^2+y^2)$

and therefore $u_z = -\frac{\rho g}{E}\left(lz - \frac{z^2}{2}\right) + \frac{\sigma\rho g}{2E}(x^2+y^2)$

$$= -\frac{\rho g}{2E}\left[l^2 - (l-z)^2 - \sigma(x^2+y^2)\right]$$