Problem 1.

"Express the mutual capacity C of two conductors (with charges $\pm e$) in terms of the coefficients C_{ab}."

The mutual capacity is defined by $e = C(\phi_2 - \phi_1)$.

To make progress we will write the potentials in terms of charges: $\phi_a = \sum_b C^{-1}_{ab} e_b$.

$$\begin{cases} \phi_1 = C^{-1}_{11} e_1 + C^{-1}_{12} e_2 \\ \phi_2 = C^{-1}_{12} e_1 + C^{-1}_{22} e_2 \end{cases} \sim \begin{cases} \phi_1 = e[C^{-1}_{11} - C^{-1}_{12}] \\ \phi_2 = e[C^{-1}_{12} - C^{-1}_{22}] \end{cases}$$

Hence, we get that:

$$-\cancel{e} = C(\cancel{e}[C^{-1}_{12} - C^{-1}_{22}] - \cancel{e}[C^{-1}_{11} - C^{-1}_{12}])$$

$$\Longleftrightarrow$$

$$\frac{1}{C} = -(2C^{-1}_{12} - C^{-1}_{11} - C^{-1}_{22}).$$

Here we could write the C^{-1}_{ab} terms in terms of C_{ab} to make it clearer.

Generally, $\begin{pmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{pmatrix}^{-1} = \frac{1}{C_{11}C_{22} - C_{12}^2} \begin{pmatrix} C_{22} & -C_{12} \\ -C_{12} & C_{11} \end{pmatrix}$

where $C^{-1}_{11} = \frac{C_{22}}{C_{11}C_{22} - C_{12}^2}$, $C^{-1}_{12} = \frac{-C_{12}}{C_{11}C_{22} - C_{12}^2}$, $C^{-1}_{22} = \frac{C_{11}}{C_{11}C_{22} - C_{12}^2}$

s.t. $\frac{1}{C} = -\left(\frac{-2C_{12} - C_{22} - C_{11}}{C_{11}C_{22} - C_{12}^2}\right) = \boxed{\frac{C_{11} + 2C_{12} + C_{22}}{C_{11}C_{22} - C_{12}^2}}$