1.5 Problem 5

Two non-interacting neutrons are in a potential $U = \frac{1}{2}m\omega^2 r^2$ in magnetic field H. What should be H for given ω in order for the ground state to have spin 0?

A neutron has spin, and because of this, it also has a magnetic moment $\boldsymbol{\mu}$. A tiny magnet in an external magnetic field has energy $U = -\boldsymbol{\mu} \cdot \boldsymbol{H}$. If the magnetic moment points along the field, then we have a lower energy, indicated by the minus sign.

For $S = 0$ to be the ground state, the energy of the singlet state needs to be less than the energy of the triplet state. The contributions to their energies consists of the harmonic potential and magnetic parts. The energy levels for a harmonic oscillator is $E_n = \hbar\omega(N + 3/2)$, where $N = 1, 2, 3...$ etc. The spin wave function for $S = 0$ is antisymmetric, thus the spatial part is symmetric, i.e., both neutrons can be in the ground state, each with energy $E_0 = \frac{3}{2}\hbar\omega$. For $S = 1$, the spatial wave function needs to be antisymmetric such that the energies are $E_0 = \frac{3}{2}\hbar\omega$ for the one neutron and $E_1 = \frac{5}{2}\hbar\omega$ for the other neutron.

Regarding the magnetic part, evidently for $S = 0$, the total magnetic moment is zero. However, for $S = 1$, we need to consider that the lowest energy is when the magnetic moment is aligned with the magnetic field, i.e., $\cos\theta = 0 \implies U = -\mu H$. But, there are two neutrons, so the total contribution from the magnetic potential is $-2\mu H$. (The triplet has three spin components but $M = -1$ is the lowest one since $\gamma < 0$.)

Comparing the energies:

$$E_{S=0} < E_{S=1} \implies 3\hbar\omega < 4\hbar\omega - 2\mu H \implies H < \frac{\hbar\omega}{2\mu} \tag{1}$$