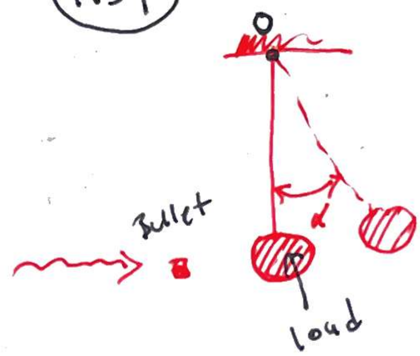
7.31



$$\vec{M}_0^{\text{before}} = \vec{M}_0^{\text{after}}$$

For the total system (bullet & load), the rate of change of angular momentum about O equals the moment of external forces about O, i.e., $d\vec{M}_0/dt = \vec{K}_0$, where $\vec{K}_0 = \sum_a \vec{r}_a \times \vec{F}_a$. (From §9.)

We choose O at the suspension point since the external forces have zero moment during the collision. From $d\vec{M}_0/dt = 0$ it follows that $\vec{M}_0 =$ constant during the collision. Importantly, AM is not conserved about O throughout the entire pendulum motion since once the load moves away from the bottom, gravity is no longer parallel to $\vec{r}$. We only conserve it immediately before & after impact.

For our two particle system,

$$\vec{M}_0 = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$$

(AM is additive and depends on the choice of origin)

Choosing y-axis positively downward, then $|\vec{M}_0^{\text{before}}| = m_1 v l$ and $|\vec{M}_0^{\text{after}}| = m_1 u_1 l + m_2 u_2 l$. Thus, $M_0^{\text{before}} = M_0^{\text{after}}$ is

$$m_1 v l = m_1 u_1 l + m_2 u_2 l.$$

(1.31)

After the collision, the load's position is sufficiently specified by the angle α. Secondly, by energy conservation we have

$$\frac{1}{2} M u_2^2 = M g l (1 - \cos \alpha)$$

The insight of §6 is to use energy when solving for the full motion, given a time-independent conservative stage. Solving for α from the energy equation gives

$$\alpha(u) = \arccos\left(1 - \frac{u^2}{2gl}\right)$$

In a contorted way, as u increases, the argument of arccos decreases, so the angle increases. Let's check each case now. Case 1: bullet pierces the load. From the AM conservation eq. $m_1 v l = m_1 u_1 l + m_2 u_2 l \Rightarrow u_2 = \frac{m_1}{m_2}(v - u_1)$

Case 2: bullet sticks in the load. Bullet & load now move with common velocity u_s s.t. $m_1 v l = (m_1 + m_2) u_s l$ so

$u_s = \frac{m_1 v}{m_1 + m_2}$. Case 3: bullet recoils. The AM conservation is $m_1 v l = m_1(-u_1) l + m_2 u_2 l \Rightarrow u_2 = \frac{m_1}{m_2}(v + u_1)$.

Clearly recoil velocity > sticking velocity > piercing velocity.

1.31

We conclude that the smallest angle is when the bullet pierces the load, intermediate angle is when the bullet strikes and finally the greatest angle is when the bullet recoils. (In agreement with Sena.)

Emphasized here is solving motion by starting from conservation laws.